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NOTE

Evaluation of a Simplified Hueckel Edge-Line Detector

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The Hueckel edge detector is one of a class of edge detectors which operate by approximating the image input by a truncated series of orthogonal functions. The number of terms in the expansion determines the accuracy of approximation and also the complexity of the process. In this paper, effects of reducing the terms used for expansion are examined. Experimental results indicate a significant loss in performance by such simplifications.

1. INTRODUCTION

An edge detector due to Hueckel [1, 2] has been in wide use. Though no definitive studies have been made, its performance is considered to be superior to that of many other edge detectors (e.g., see [3]; see [4] for a contrary conclusion). However, the computation required for the Hueckel edge detector is considerably more than that for other types of operators. The central part of the Hueckel edge detector involves approximation of a two-dimensional image intensity pattern by expansion into a truncated Fourier series. Mero and Vassy have described a method of simplifying the approximation by using fewer bases functions for expansion [5]. In this paper, we examine a generalization of this approach and the effect on the performance of a reduction in the number of basis functions. The main conclusion is that such reduction in computation is achieved at a significant loss in performance, if the images in use are noisy.

2. OPERATION OF THE HUECKEL EDGE-LINE DETECTOR

The basic process of the Hueckel edge operator consists of optimally fitting an ideal edge-line to the image intensity values in a small circular neighborhood. The ideal edge-line is determined by a 6-tuple of parameters, three parameters determine the brightness levels (b_-, t_-, and t_+, as shown in Fig. 1) and the other three parameters determine the position, orientation, and width of the line. The fitting process consists of determining the values of the six parameters for a best fit with the image intensities. Ideally a tuple of parameters is to be computed such that

$$N = \|I - S\ (\text{tuple})\| \tag{1}$$

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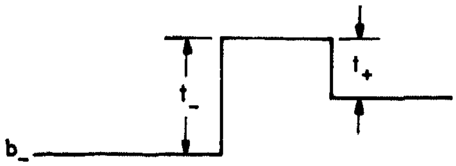

FIG. 1. Intensity profile of an ideal edge-line.

is a minimum, where I is a vector representing image intensities and S is an ideal edge-line.

This minimization process is approximated by expansion of both the input image disk and the edge-line in an orthogonal Fourier series. Let a_i be the coefficients of expansion for the image and let s_i be the coefficients for an ideal edge-line. The minimization of (1) is then approximated by choosing a tuple such that

$$N = \sum_{i=0}^{8} (a_i - s_i)^2 \tag{2}$$

is minimized. Hueckel gives arguments for the optimality of the chosen series [2]. A crucial decision is to use only the first nine terms of expansion. Two reasons are given for the choice of this number:

(a) Higher-order terms correspond to noise in the image and should be ignored.

(b) An analytical solution to the minimization problem is found using nine terms.

The decision as to the presence of an edge is based on the amplitude of the computed step, as well as the degree of fit, determined by N in Eq. (1) above.

3. SIMPLIFICATION OF THE OPERATOR

The computational cost of the Hueckel operator can be reduced substantially if the number of terms used in the expansion is reduced. An edge-line is completely specified by six parameters. Further, the absolute brightness level (b_-) is of little interest for edge computation. Six coefficients of expansion suffice for determining the other five parameters. Six, rather than five, coefficients are needed as some coefficients have a zero value for certain directions of an edge. Also, the problem of minimization in Eq. (2) reduces to that of determining five unknowns from six equations (the details are in the Appendix).

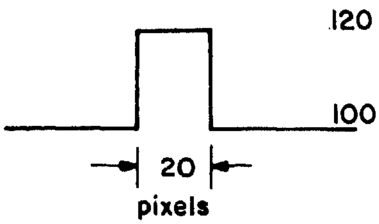

FIG. 2. Intensity profile of the test step (without noise).

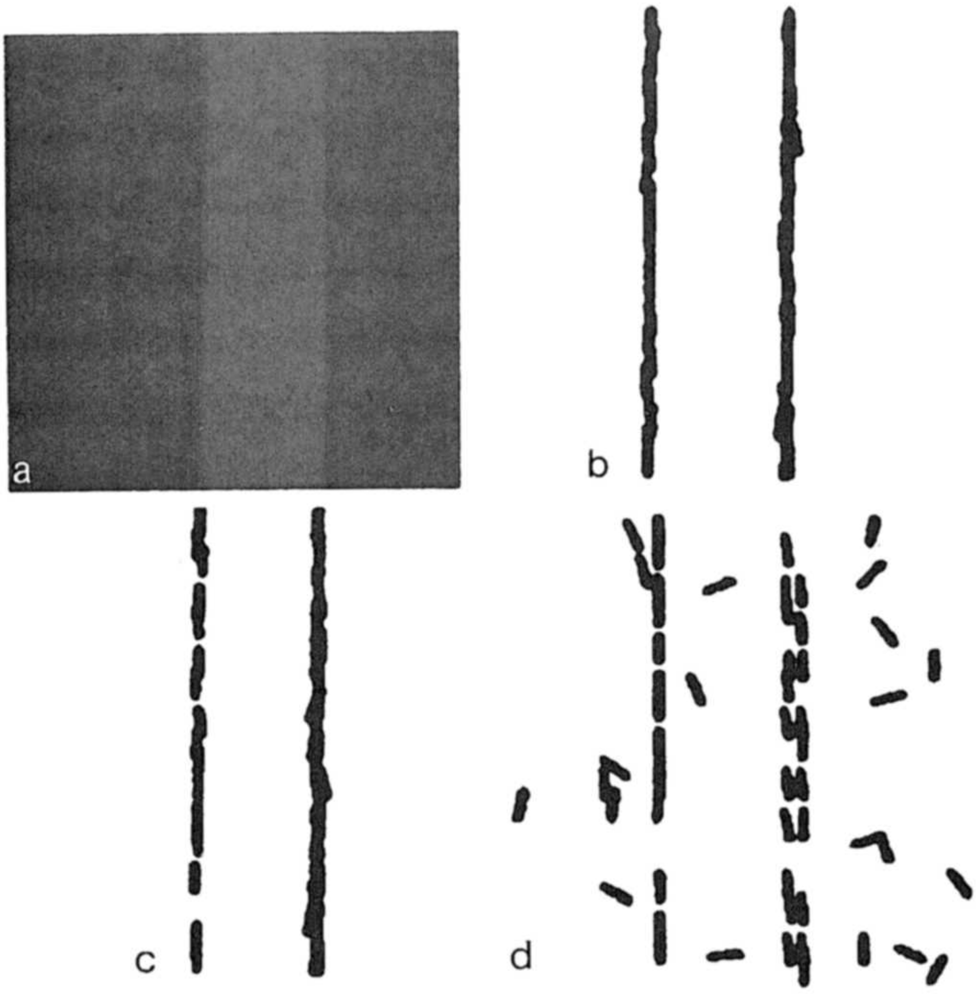

FIG. 3. A low-noise image and detected edges.

Only three parameters (specifying orientation, position, and step size) are needed if only the step edges are of interest. Mero and Vassy describe an approach to determining these parameters for a square window of the image [5]. They conclude that expansion in only two bases functions is needed to determine the orientation of the step. The position and step size are then determined by an approximation.

We have generalized the procedure to determine the three parameters for a step from three coefficients of expansion, used by Hueckel, for a circular neighborhood and determining the five parameters for a step-line from six coefficients of expansion.

In either case, as far as the truncated expansion is concerned, the signal can be fit perfectly with an ideal edge or an ideal edge-line. Thus, no minimization computation is necessary.

The important question is the effect of ignoring some terms in the orthogonal expansion of the signal. It is clear that if the signal indeed consists of an ideal edge element, then fewer terms of expansion are sufficient to characterize the signal exactly. Results of experimental evaluation on signals containing noise are presented next.

4. EXPERIMENTAL RESULTS

The input for these experiments was a picture with two vertical edges; the intensity profile was as illustrated in Fig. 2. A varying amount of random, Gaussian noise was added to this image. Results are presented for three images, Figs. 3a, 4a, and 5a, with step size-to-noise variance ratios of 10, 4, and 2.

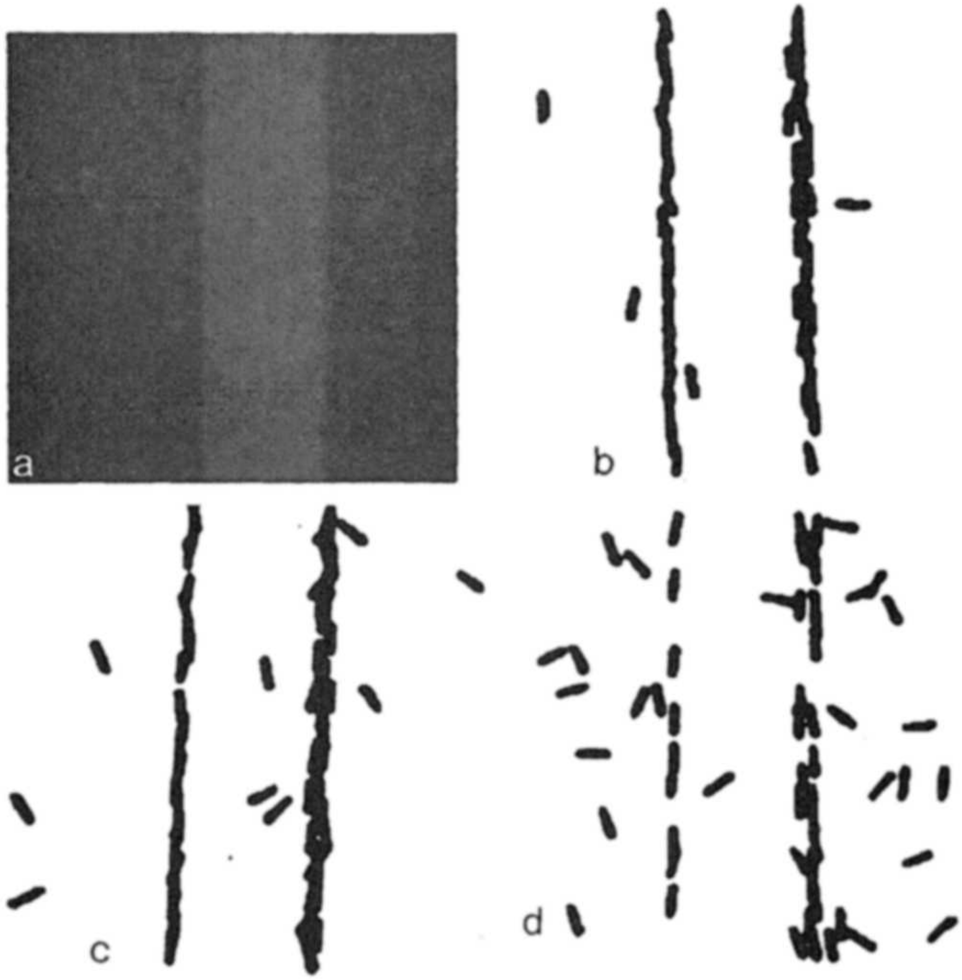

FIG. 4. A medium-noise image and detected edges.

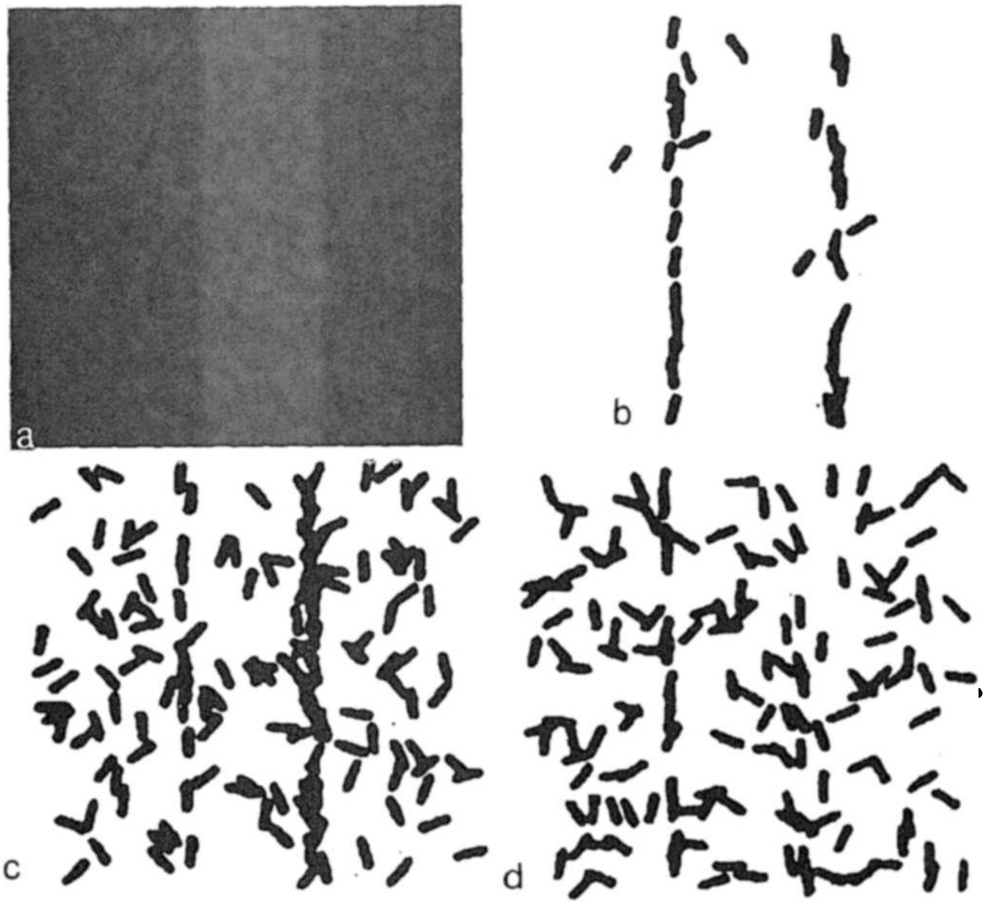

FIG. 5. A high-noise image and detected edges.

The three images were processed by three versions of the edge detector:

1. Original Hueckel edge detector (operator 1),
2. Edge-line detector using six coefficients (operator 2),
3. Edge detector using three coefficients (operator 3).

Determining the presence of an edge requires the use of a threshold on the computed step amplitude. Use of a higher threshold will allow fewer noise edges but also lose some of the desired edges. For the results presented, the thresholds were determined for the best subjective performance. The differences in the performance of the three operators are so marked that a more careful control of thresholds (e.g., by requiring the total number of detected edges to be the same) was not considered necessary.

Figures 3, 4, and 5 show the results of applying the three operators to the three images. (The edge detector was applied to every third pixel along every third row. Each application looks at a neighborhood that is approximately circular and eight pixels in diameter. There is considerable overlap of regions in two neighboring applications, resulting in multiple edges for the same step. This phenomenon is pronounced in Figs. 3, 4, and 5 for the right edge only, because of accidental placements.)

It is clear that the performance of operator 1 declines gradually with the increase in added noise. Operators 2 and 3, on the other hand, perform reasonably well for high signal-to-noise ratio, but very poorly for images with high noise. Further, the performance of operator 2, using more coefficients of expansion, is superior to that of operator 3. (In the absence of any noise, all three operators perform perfectly.)

Similar results are obtained for detection of line edges (i.e., when the width of the step is small, say two or three pixels wide).

5. CONCLUSIONS

The results shown above are somewhat surprising. It is tempting to think that since the ideal step is completely determined by three parameters, three bases vectors can be found to span the space of ideal step elements. Unfortunately, the space spanned by the ideal steps is not a complete space (the addition of two arbitrary steps is not another step). (For the special case of edges defined on a 3×3 square neighborhood, a complete two-dimensional subspace spanned by the edges has been determined by Frei and Chen [6].)

It is concluded that simplification of the Hueckel edge detector by ignoring some of the terms of the Fourier expansion results in unacceptable loss in performance, if the images are likely to be noisy. (Mero and Vassy present no results on comparable images in their claim that the performance is not compromised.) Further, this raises questions as to the effect of using only nine terms in Hueckel's original operator and also the effects of similar approximations using other series for expansion (e.g., see [7]).

APPENDIX: DETAILS OF THE APPROXIMATIONS

Familiarity with Refs. [1, 2] is assumed here and the notation used therein is used here without much elaboration. The nine coefficients of expansion for an ideal edge-element are as follows:

$$s_0 = (3\pi)^{\frac{1}{2}}(32b_- + t_-(16 - 21r_- + 7r_-^3 - 7r_-^5 + 5r_-^7) + t_+(16 - 21r_+ + 7r_+^3 - 7r_+^5 + 5r_+^7)/56, \quad (3)$$

$$s_1 = \lambda_- r_- + \lambda_+ r_+, \quad (4)$$

$$s_2 = (\lambda_- + \lambda_+)c_x, \quad (5)$$

$$s_3 = (\lambda_- + \lambda_+)c_y, \quad (6)$$

$$s_4 = 2^{\frac{1}{2}}(\lambda_- r_- + \lambda_+ r_+)(c_x^2 - c_y^2), \quad (7)$$

$$s_5 = 8^{\frac{1}{2}}(\lambda_- r_- + \lambda_+ r_+)c_x c_y, \quad (8)$$

$$s_6 = 5^{\frac{1}{2}}(\lambda_- r_-^2 + \lambda_+ r_+^2)c_x, \quad (9)$$

$$s_7 = 5^{\frac{1}{2}}(\lambda_- r_-^2 + \lambda_+ r_+^2)c_y, \tag{10}$$

$$s_8 = \lambda_- r_-(0.5 - 2.5r_-^2) + \lambda_+ r_+(0.5 - 2.5r_+^2), \tag{11}$$

where r_-, r_+ stand for the positions of brightness transitions, b_-, t_-, and t_+ are as shown in Fig. 1, $\lambda_\pm = (3\pi)^{\frac{1}{2}} t_\pm (1 - r_\pm^2)^2/4$, and c_x and c_y are the direction cosines of the edge orientation (see [1] for details). Consider two cases separately.

1. Step edges only. The ideal edge-line can be converted to a step by setting $t_+ = 0$ (hence $\lambda_+ = 0$). We are now interested in determining λ_-, r_-, c_x and c_y. We need use only Eqs. (4), (5), and (6) and hence need only three coefficients.

2. Step and line edges. Six parameters need to be determined now and we need six equations. Several subsets of the above nine equations will suffice. In particular, Eqs. (4) through (9) are sufficient. The solution can proceed by determining c_x and c_y from Eqs. (5) and (6) (note that $c_x^2 + c_y^2 = 1$). Parameters λ_-, λ_+, r_-, and r_+ can now be determined from four equations: (4), (5) or (6), (9) or (10), and (11). These equations can be solved in exactly the same way as those in Ref. [1, Appendix A]. (Note that computation of c_x and c_y using all nine coefficients requires iterative solution of a fourth-order polynomial.)

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