Geodesic distance evolution of surfaces: a new method for matching surfaces

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Abstract

The general problem of surface matching is taken up in this study. The process described in this work hinges on a geodesic distance equation for a family of surfaces embedded in the graph of a cost function. The cost function represents the geometrical matching criterion between the two 3D surfaces. This graph is a hypersurface in 4-dimensional space, and the theory presented herein is a generalization of the geodesic curve evolution method introduced by R. Kimmel et al [12]. It also generalizes a 2D matching process developed in [4]. An Eulerian level-set formulation of the geodesic surface evolution is also used, leading to a numerical scheme for solving partial differential equations originating from hyperbolic conservation laws [17], which has proven to be very robust and stable. The method is applied on examples showing both small and large deformations, and arbitrary topological changes.

1. Introduction and previous work

The problem of matching structures is a challenging issue in computer vision and graphics. It has a large number of applications and the definition of structures characteristics depends on the problem addressed.

A general matching formulation problem can be stated as: given two structures $\mathcal{S}$ and $\mathcal{D}$ define a function which associates to any point of $\mathcal{S}$ a corresponding point on $\mathcal{D}$. The characterization of such function is an ill-posed problem in the sense there is no unique solution. However, introducing structures properties such as the geometry or the underlying image representation allows to characterize a unique matching function. Commonly used features are pixels grey level values for stereoscopic matching or optical flow [5], edges for token based approaches [21] and geometric properties of the structures [22, 23]. The latter properties are more robust since they can deal with situations where there is no consistency of the image grey level value. Relevant geometric properties are selected on the basis of their ability to characterize a description of the structures which is invariant to the actual deformation. In the case of rigid or small elastic deformations high curvature points [3, 16] or semi-differential invariants [14] can be considered as an invariant description of the structure. Higher order geometric description can also be considered such as crest and ridge lines in order to characterize 3D structures properties [19]. These methods perform well but cannot deal large deformation or when we are interested in studying the evolution of a structure whose topology evolves in time [4]. This situation is very common in computer graphics where we are interested in defining a morphing function, *ie.* a set of paths departing from the source $\mathcal{S}$ and ending at the destination structure $\mathcal{D}$. Here the matching criterion translates into geometric constraints on the paths themselves [15]. Matching features in 3D images is also an important task in medical image analysis [8, 20].

In this paper the general problem of matching two arbitrary surfaces $\mathcal{S}$ and $\mathcal{D}$ in $\mathbb{R}^3$ is contemplated. The method proposed represents a generalization of the algorithm introduced by Cohen *et al* [4] for curves matching. This scheme has also motivated a generalization of the geodesic curve propagation method introduced by R. Kimmel *et al* [12] into a surface evolution framework. First, we briefly summarize the 2D case (section 2) in order to clearly state the objectives for 3D surfaces matching. As the curve matching is based on the curve evolution on a surface in $\mathbb{R}^3$, we set up a generalization by considering a geodesic surface evolution scheme on a hypersurface embedded in $\mathbb{R}^4$ (see section 3.3). This theory makes use of the Hodge "star" ($*$) operator (briefly presented in subsection 3.2). Subsection 3.4 is devoted to the level-set formulation of that geodesic surface evolution scheme, allowing for the computation of distance maps on the cost hypersurface. The level-set evolution equation takes the form of Hamilton-Jacobi partial differential equations, for which J. Sethian [17] has introduced stable and robust numerical resolution schemes. The surface matching method is then introduced. It consists in computing the orbits of a vector field defined from the gradients of the distance maps. These matching paths between $\mathcal{S}$ and $\mathcal{D}$ minimize a cost function. Different matching criteria are presented in section 4, leading to various definitions of the cost hypersurface. The method is applied on various examples, and the paper ends with conclusion and perspectives.

2. Curve Matching

In this section, the curve matching approach proposed by I. Cohen *et al.* [4] is reminded. Let $\mathcal{S}$ and $\mathcal{D}$ be two plane curves. The matching of $\mathcal{S}$ and $\mathcal{D}$ is computed by deriving a cost hypersurface $W \subset I\!R^3$ where $\mathcal{S}$ and $\mathcal{D}$ are lying. W measures the similarity of the two structures. Distance maps on W are determined from a specific curve evolution scheme on W. Lastly, minimal paths between $\mathcal{S}$ and $\mathcal{D}$ are computed on the surface W.

Let us examine more closely the curve evolution scheme on W. In [12], it is proven that if $\mathcal{S} = \alpha(u, 0)$ is an initial curve parameterized by u, and lying on a surface in $I\!R^3$, the curve $\alpha(u, t)$ drawn on the same surface, and whose every point is at geodesic distance t from $\mathcal{S}$ is solution of the following partial differential equation:

$$\frac{\partial \alpha}{\partial t} = \vec{N} \otimes \vec{\tau^u}, \tag{1}$$

where $\vec{\tau^u}$ is the unitary tangent vector to α: $\vec{\tau^u} = \frac{\partial \alpha}{\partial u} / || \frac{\partial \alpha}{\partial u} ||$, $\vec{N}$ the normal to the surface, and the symbol $\otimes$ stands for the cross-product of two vectors in $I\!R^3$.

As any attempt at solving equations (1) may turn out to be quite difficult in general (the curve is constrained to remain on a surface) Kimmel *et al.* [12] make use of a property coming from the front propagation theory, and simplify the problem by studying the normal speed of the projected curve $\pi \circ \alpha(u, t)$ where $\pi : I\!R^3 \to I\!R^2$ denotes the standard projection onto the (x, y) plane. For that matter, the surface W on which the curves are lying is supposed to be a graph surface, *ie.* the graph of a function $z(x, y)$. It is shown that the normal speed V of the projected curves is given by the formula:

$$\begin{aligned} V &= \sqrt{\frac{n_1^2(1+q^2) + n_2^2(1+p^2) - 2pqn_1n_2}{(1+p^2+q^2)}} \\ &= \sqrt{an_1^2 + bn_2^2 - cn_1n_2} \end{aligned} \tag{2}$$

with $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$, and (n_1, n_2) the coordinates of the normal vector to the projected curve. Kimmel *et al.* also show that, if the projected curve is written using an Eulerian approach, as a level-set $\varphi^{-1}(0)$, the associated evolution equation is:

$$\varphi_t = \sqrt{a\varphi_x^2 + b\varphi_y^2 - c\varphi_x\varphi_y}. \tag{3}$$

(with $\varphi_t = \frac{\partial \varphi}{\partial t}$, *etc*) for which stable and powerful numerical resolution schemes have been proposed [17].

In [4], these results are used to build a curve matching process. Using the same formulation for $\mathcal{D} = \beta(u', 0)$, and considering a level-set formulation of the projection $\pi \circ \beta = \psi^{-1}(0)$, the authors set up a cost function incorporating geometric matching criteria (position and curvature) and compute the path γ of a matching function by integrating the differential equation:

$$\frac{\mathrm{d}\gamma}{\mathrm{d}s} = -\nabla(\varphi + \psi) \tag{4}$$

with φ and ψ being the two solutions of equation (3).

In the next sections we generalize this curve matching method to the case of surfaces.

3. Surface Matching

Using the same approach, one can generalize the matching process to surfaces. We keep the same notations as used in the previous section, but each object has an additional dimension. $\mathcal{S}$ and $\mathcal{D}$ are two surfaces to be matched. W is a cost hypersurface embedded in $I\!R^4$, it represents the matching criteria. $\mathcal{S}$ and $\mathcal{D}$ are lying on W.

The matching process still follows the same scheme, we have to:

1. build a cost hypersurface W between $\mathcal{S}$ and $\mathcal{D}$,
2. compute, using a level-set formulation, the evolution of a surface propagation scheme on the hypersurface W between $\mathcal{S}$ and $\mathcal{D}$,
3. use the geodesic evolution scheme to generate distance maps on W,
4. find the paths of minimal costs on W between $\mathcal{S}$ and $\mathcal{D}$.

The main difficulty of this extension is the generalization of the result expressed by R. Kimmel *et al.* [12], *ie.* the generalization of equation (1) and its level-set formulation (3). The following sections are devoted to this generalization.

3.1. Definitions

We suppose the hypersurface W compact and pathwise connected[1]. From these assumptions, one can derive, using

[1] These assumptions do not restrict the validity of the theory presented in this work, as W will appear as the graph of a cost function, which automatically satisfies these requirements in practice.

an easy application of the Hopf-Rinow-De Rham theorem (see [18]) that given any two points M_0 and $M_1 \in W$, there is a unique path $\gamma : [0,1] \rightarrow W$ connecting M_0 and M_1 ($\gamma(0) = M_0$, $\gamma(1) = M_1$) whose length minimizes the lengths of all paths connecting the two points. The length of γ is called the geodesic distance between M_0 and M_1, and will be denoted $d_W(M_0, M_1)$. Moreover, the path γ is necessarily a geodesic curve on W (*ie.* a curve such that the second derivative $\dfrac{\mathrm{d}^2\gamma}{\mathrm{d}u^2}$ is always perpendicular to W).

Let $\mathcal{S} \subset W$ (resp. $\mathcal{D} \subset W$) an initial surface defined by a local parameterization $\mathcal{S} = \alpha(u,v,0)$ (resp. $\mathcal{D} = \beta(u',v',0)$). We consider a local parameterization $\alpha(u,v,t)$ (resp. $\beta(u',v,'t)$) of the surface in W whose points are located at geodesic distance t from $\mathcal{S}$ (resp. $\mathcal{D}$):

$$\begin{array}{c}\alpha(u,v,t) = \{M \in W \mid d_W(M,\mathcal{S}) = t\} \\ (\text{resp. } \beta(u',v',t) = \{M \in W \mid d_W(M,\mathcal{D}) = t\}).\end{array} \quad (5)$$

We are interested in defining a partial differential equation governing the evolution of the two surfaces as the parameter t evolves. For this purpose, we need a notion of cross-product in 4-space, and a method of deriving simple formulae about such a cross-product. The mathematical tool that can achieve these requirements is given by the Hodge $*$ operator, a notion recalled in the next subsection.

3.2. The Hodge $*$ operator

The theory is only briefly reviewed here. The reader is referred to [1] and [11] for a complete presentation of the subject. Let $\Lambda^p(I\!R^n)$ be the p-exterior power of space $I\!R^n$ and $*$ the Hodge operator defined, for every $\lambda \in \Lambda^p(I\!R^n)$ by the equality:

$$\lambda \wedge \mu = \langle *\lambda, \mu \rangle_{n-p}\, e_1 \wedge e_2 \wedge \cdots \wedge e_n \quad (6)$$

In equation (6) $(e_1, \cdots, e_n)$ is the standard basis of $I\!R^n$, $\langle\ ,\ \rangle$ is the usual dot product on $\Lambda^p(I\!R^n)$ defined on generators by:

$$\langle u_1 \wedge \cdots \wedge u_p, w_1 \wedge \cdots \wedge w_p \rangle_p = \det(\langle u_i, w_j \rangle) \quad (7)$$

and $\mu \in \Lambda^{n-p}(I\!R^n)$. From the property $*(*\lambda) = (-1)^{p(n-p)}\lambda$, one can deduce that, if u, v and w are three linearly independent vectors in $I\!R^4$, the associate image $*(u \wedge v \wedge w) \in \Lambda^1(I\!R^4) = I\!R^4$ satisfies the following properties:

- it is a vector in $I\!R^4$ perpendicular to u, v and w.
- The basis $(u, v, w, *(u \wedge v \wedge w))$ is positively oriented.
- Its squared norm : $\| *(u \wedge v \wedge w) \|^2$ is equal to $\begin{vmatrix} \langle u,u\rangle & \langle u,v\rangle & \langle u,w\rangle \\ \langle v,u\rangle & \langle v,v\rangle & \langle v,w\rangle \\ \langle w,u\rangle & \langle w,v\rangle & \langle w,w\rangle \end{vmatrix}$.

Using the Hodge $*$ operator, we will derive the geodesic distance evolution scheme for the two families of surfaces $\alpha(u,v,t)$ and $\beta(u,v,t)$. This is presented in the following subsection.

3.3. The geodesic distance evolution equation

From this point on, we consider the local parameterization $\alpha(u,v,t)$ of surface $\mathcal{S}$, and the evolution scheme will apply to $\beta(u',v',t)$ as well.

It is a known result that an *orthogonal* parameterization can always be found (see [6] p. 183). Let us consider such a parameterization, *ie.* such that $\langle \vec{\tau^u}, \vec{\tau^v} \rangle = 0$, with $\vec{\tau^u} = \dfrac{\alpha_u}{\| \alpha_u \|}$ and $\vec{\tau^v} = \dfrac{\alpha_v}{\| \alpha_v \|}$.

Let $\vec{N}$ the normal vector to W. It can be proven that if $\alpha(u,v,t)$ (the local and orthogonal parameterization of a family of surfaces indexed by t) satisfies the following equation:

$$\frac{\partial \alpha}{\partial t} = *\left(\vec{N} \wedge \vec{\tau^u} \wedge \vec{\tau^v}\right), \quad (8)$$

then $\alpha(u,v,t)$ is, for each t, a local and orthogonal parameterization of:

$$\alpha(u,v,t) = \{M \in W \mid d_W(M,\mathcal{S}) = t\},$$

which is the surface in W whose points are located at geodesic distance t from $\mathcal{S}$.

The whole proof can be expressed in two steps [10, 11], according to the proof given in [12]. We do not give this proof here, but just the idea. First, we have to prove, that if equation (8) is verified then the curves $\alpha(u_0, v, 0_t)$, such that $u = u_0$ and $v = v_0$ are fixed, are geodesic curves of

Lemma 1 *Let $\gamma(t)$ be the curve in W defined by:*

$$\gamma(t) = \alpha(u,v,t)|_{u=u_0, v=v_0,\ \text{fixed}}.$$

For any u_0, v_0, the curve $\gamma(t)$ is a geodesic in W.

Then, it is possible to use the Gauss lemma to prove that $\alpha(u,v,t)$ is necessarily a local parameterization of a family of surfaces at geodesic distance from $\mathcal{S}$. Hence:

Lemma 2 *The evolution of the family of surfaces $\alpha(u,v,t)$ is given by the equation*

$$\frac{\partial \alpha}{\partial t} = *\left(\vec{N} \wedge \vec{\tau^u} \wedge \vec{\tau^v}\right). \quad (9)$$

From Lemma 2, we have derived a geodesic distance evolution scheme for a family of surfaces described by local parameterizations. In the next subsection, we compute the normal speed of the projection of $\alpha(u,v,t)$ (resp. $\beta(u',v',t)$) onto the (x,y,z) hyperplane of $I\!R^4$.

3.4. Normal evolution of the projection and resolution of the equation

To generalize the 2D case, we now make the assumption that W is a graph hypersurface, that is to say $W = \{(x, y, z, w(x, y, z))\}$ for a function $w : I\!R^3 \to I\!R$. Let $\pi : I\!R^4 \to I\!R^3$ be the canonical projection onto the (x, y, z) hyperplane in $I\!R^4$. Let $\mathcal{S}(t)$ (resp. $\mathcal{D}(t)$) be the projection of $\alpha(u, v, t)$ (resp. $\beta(u', v', t)$) onto (x, y, z):

$$\mathcal{S}(t) = \pi \circ \alpha \ \ (\text{resp. } \mathcal{D}(t) = \pi \circ \beta).$$

We denote by p, q, r the following quantities: $p = \dfrac{\partial w}{\partial x}$, $q = \dfrac{\partial w}{\partial y}$ and $r = \dfrac{\partial w}{\partial z}$. Starting from a result mentioned in [7], we admit that the trace of a propagating surface may be determined only by its normal velocity, as the other components of the velocity influence only the local parameterization. Our goal is then to compute the projected velocity of the evolving surface $V = \langle \pi \circ \alpha_t, \vec{n} \rangle$, $\vec{n} = (n_1, n_2, n_3)$ being the normal to the projected surface $\pi \circ \alpha(u, v, t)$. One can prove the following:

Lemma 3 *The projected surface $\mathcal{S}(t)$ satisfies the normal propagation rule:*

$$\frac{\partial \mathcal{S}}{\partial t} = V\vec{n} \tag{10}$$

with

$$V = \sqrt{an_1^2 + bn_2^2 + cn_3^2 - dn_1n_2 - en_1n_3 - fn_2n_3} \tag{11}$$

with:

$$\begin{aligned} a &= \frac{1 + q^2 + r^2}{1 + p^2 + q^2 + r^2}, & b &= \frac{1 + p^2 + r^2}{1 + p^2 + q^2 + r^2} \\ c &= \frac{1 + p^2 + q^2}{1 + p^2 + q^2 + r^2}, & d &= \frac{2pq}{1 + p^2 + q^2 + r^2} \\ e &= \frac{2pr}{1 + p^2 + q^2 + r^2}, & f &= \frac{2qr}{1 + p^2 + q^2 + r^2} \end{aligned} \tag{12}$$

The proof can be read in [9]. It comes from equation (6) and simplifications given by the choice of an orthogonal parameterization.

Having found the normal equation evolution of the projected surface, we now proceed to set up an Eulerian formulation for $\mathcal{S}(t)$ and $\mathcal{D}(t)$, by writing these projected surface as level-sets $\varphi^{-1}(0)$ and $\psi^{-1}(0)$, with $\varphi : I\!R^3 \to I\!R$ and $\psi : I\!R^3 \to I\!R$. This idea was introduced by Osher and Sethian [13] for crystal growth modelling. Its major advantage is the ability to handle topological changes and singularities while insuring stability and accuracy. Moreover, this level-set formulation gives us the ability to compute distance maps on the graph of the cost function. Generalizing the 2D case (see equation (3)), one sees that the function φ follows the propagation equation[2]:

$$\varphi_t = \sqrt{a\varphi_x^2 + b\varphi_y^2 + c\varphi_z^2 - d\varphi_x\varphi_y - e\varphi_x\varphi_z - f\varphi_y\varphi_z} \tag{13}$$

As mentioned in the curve evolution process described in [12] such an Eulerian formulation leads to numerical resolution schemes able to handle problems caused by a time varying coordinate system (u, v, t): curvature singularities and topological changes. These kind of resolution methods called $minmod$ approaches are well described in [17]. These explicit schemes are known to be stable and robust.

Using these, we can now use the computed functions φ and ψ to generate distance maps on W. In order to initialize the resolution scheme for computing geodesic distance maps on W described by φ and ψ, we have to define initial estimate φ_0 and ψ_0 such that the initial surfaces $\mathcal{S}$ and $\mathcal{D}$ are represented through a level-set of φ_0 and ψ_0. This initial estimate can be obtained in several ways according to the data. We use a Euclidean distance map [2] in such a way that:

$$\varphi_0(x, y, z) \ \ (resp. \psi_0(x, y, z)) =$$

$$\begin{cases} -d(x, y, z) & \text{if } (x, y, z) \text{ is inside } \mathcal{S} \text{ (resp. } \mathcal{D}) \\ 0 & \text{if } (x, y, z) \in \mathcal{S} \text{ (resp. } \mathcal{D}) \\ +d(x, y, z) & \text{if } (x, y, z) \text{ is outside } \mathcal{S} \text{ (resp. } \mathcal{D}). \end{cases} \tag{14}$$

Given W and the initial estimates φ_0 and ψ_0 on this hypersurface, the resolution scheme allows us to compute the distance from $\mathcal{S}$ and $\mathcal{D}$ as:

$$D_{\mathcal{S}} = \{(x, y, z, \varphi(x, y, z))\}$$

and

$$D_{\mathcal{D}} = \{(x, y, z, \psi(x, y, z))\}.$$

We can now use the previous theory to build a general matching process between the two surfaces $\mathcal{S}$ and $\mathcal{D}$ in $I\!R^3$. We just have to use the same approach as in the 2D case, and compute the paths γ in respect to the differential equation (4). The path joining the two surfaces are computed using a Runge-Kutta scheme by integrating this equation.

4. Matching criteria

The hypersurface graph W is chosen in order to incorporate a geometric criterion of similarity. It is chosen such that the two surfaces ${\varphi_0}^{-1}(0)$ and ${\psi_0}^{-1}(0)$ are level-sets of W. At this point one may choose to incorporate only distance information or curvature in the definition of W. A possible choice for W using only distance is given by:

$$W_1 = (x, y, z, w(x, y, z)) = (x, y, z, \min(|\varphi_0|, |\psi_0|)) \tag{15}$$

[2] It is important to note that the function φ depends not only of $(x, y, z) \in I\!R^3$, but also of the parameter t. To simplify the notations, we do not write explicitly that dependence on t, but it is important to keep it in mind.

To take into account curvature, we use a matching criterion ρ, function of the Euclidean distance d and of the relative difference between the mean curvatures $\Delta\kappa = \kappa_{\mathcal{S}} - \kappa_{\mathcal{D}}$, where $\kappa_{\mathcal{S}}$ and $\kappa_{\mathcal{D}}$ are respectively the mean curvatures computed on each points of $\mathcal{S}$ and $\mathcal{D}$. The ρ function is defined in such a way that the influence of mean curvature decreases as the Euclidean distance d increases:

$$\rho(\Delta\kappa, d) = 1 - \frac{\Delta\kappa^2}{1 + d^2\Delta\kappa^2/\sigma}$$

σ being a scale parameter defining the neighborhood around the two initial surfaces where mean curvature is taken into account. This leads to:

$$W_2 = (x, y, z, \min(|\varphi_0|\rho(\Delta\kappa, \varphi_0), |\psi_0|\rho(\Delta\kappa, \psi_0))) \quad (16)$$

The mean curvature is easily computed from the level-set representation of the surfaces:

$$\kappa = \frac{1}{(\varphi_x^2 + \varphi_y^2 + \varphi_z^2)^{3/2}} \times$$

$$[(\varphi_{yy} + \varphi_{zz})\varphi_x^2 + (\varphi_{xx} + \varphi_{zz})\varphi_y^2 + (\varphi_{xx} + \varphi_{yy})\varphi_z^2$$
$$-2\varphi_x\varphi_y\varphi_{xy} - 2\varphi_x\varphi_z\varphi_{xz} - 2\varphi_y\varphi_z\varphi_{yz}].$$

We give some results in the next section, using either the hypersurface definition W_1 or W_2 to express the similarity criterion.

5. Results

We apply the matching process described in the previous section on a first example illustrating the ability of the model to cope with a topological change and large deformation. The initial surface $\mathcal{S}$ is given by two spheres and the destination surface $\mathcal{D}$ is the shape of an ellipsoid containing the two spheres. On figure 1, we show the result in the computation of the matching paths, starting from the two spheres, and depicted in blue in this picture. The graph of the cost function used in this example is W_1. This example clearly proves the ability of matching dissimilar surfaces with distinct topologies. We show, in other examples displayed on figures 2 and 3, the use of the graph function W_2 incorporating the mean curvature, as in equation (16). The figure 2 corresponds to a synthetic case, matching two surfaces before and after a small deflation. In figure 3 the initial surface $\mathcal{S}$ is given by a digital elevation model of mount Etna, and the destination surface is computed by applying a geophysical deformation model to $\mathcal{S}$. This geophysical model is used to compute possible deformations observed in volcanic regions. The source surface $\mathcal{S}$ is depicted in blue, and the destination surface $\mathcal{D}$ is represented with transparency.

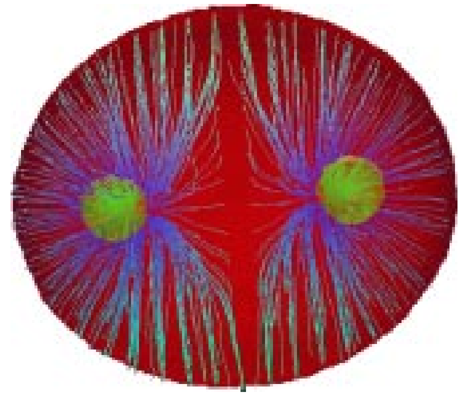

Figure 1. Matching paths between a surface made of two spheres and an ellipsoid. The paths start from the ellipsoid.

6. Conclusion

A method for solving the problem of matching two arbitrary surfaces in $I\!R^3$ is presented in this work. It uses a generalization of a geodesic distance evolution scheme for curves, and needs the development of a surface propagation equation for hypersurface in $I\!R^4$. Then, through a study of the level-set formulation for the propagation of the projected surfaces, stable numerical algorithms are used to compute distance maps on arbitrary hypersurface. These algorithms discard the drawbacks of curvature singularities and topological changes of the projected surfaces. The matching algorithm makes use these distance maps to compute optimal paths between the two surfaces. The optimal paths minimize a cost criterion which can incorporate various geometrical properties, such as distance and curvature. The matching process can serve as a basis for answering matching problems where additional information is superimposed on the original 3D-objects: for instance, a cost hypersurface may incorporate the results of motion estimation algorithms or texture information.

The matching process presented in this paper should give new insights to many problems in computer vision where the matching of complex structures is relevant.

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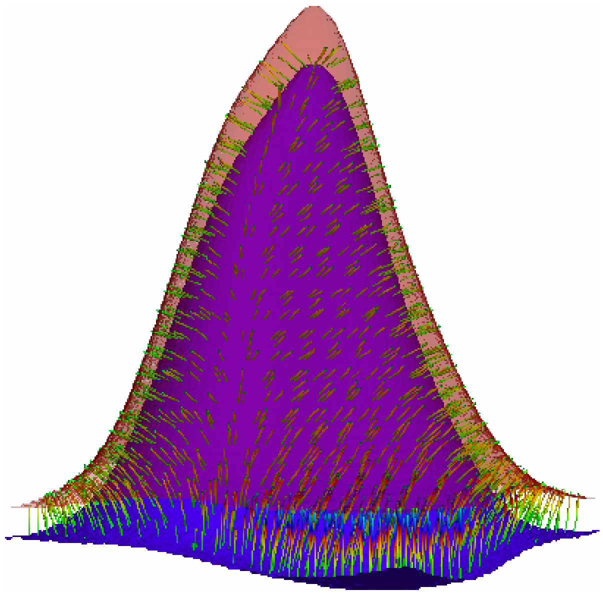

Figure 2. Matching paths between a synthetic surface before and after a small deflation.

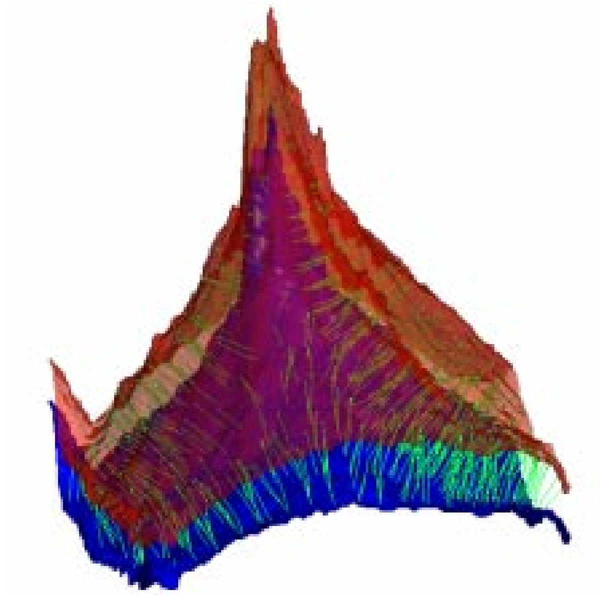

Figure 3. Matching paths between a surface obtained from a digital elevation model and a destination surface computed by a geophysical model.

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