

# Optimal Global Mosaic Generation from Retinal Images

Tae Eun Choe, Isaac Cohen, Munwai Lee, Gérard Medioni  
*IRIS, University of Southern California,*  
*{tchoe, icohen, munlee, medioni}@usc.edu*

## Abstract

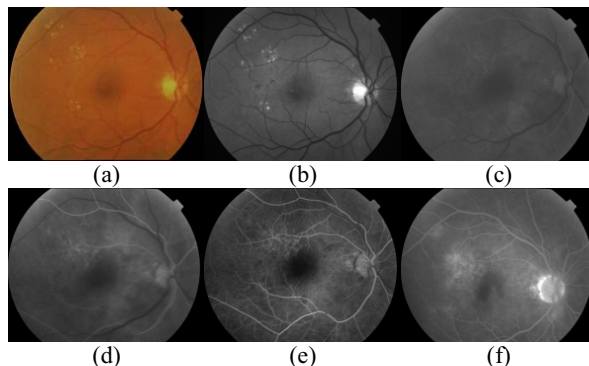
We present a method to construct a mosaic from multiple color and fluorescein retinal images. A set of images taken from different views at different times is difficult to register sequentially due to variations in color and intensity across images. We propose a method to register images globally in order to minimize the registration error and to find optimal registration pairs. The reference frame that gives the minimum registration error is found by the Floyd-Warshall's all-pairs shortest path algorithm, and all other images are registered to this reference frame using an affine transformation model. We present experimental results to validate the proposed method.

## 1. Introduction

The construction of a mosaic from multiple images is of critical importance in retinal image analysis. When examining a patient, an ophthalmologist first captures a color image of a retina using a high resolution retinal fundus camera. This is followed by a sequence of fluorescein images from the different views (because the eye is moving) at different times while the fluorescein dye goes through the blood vessels. The intensity values in the fluorescein images vary substantially during the circulation of the dye. Therefore, registration of color and fluorescein retinal images is a difficult problem. Figure 1 shows examples of the various color and fluorescein retinal images.

In order to align the images so that all features are in the same position, it is important to select invariant features for matching. Such features can be general corner point, optic disc, vessels, or the bifurcation of the vessels, so called Y-features. Among these features, Y-features are the most reliable and abundant features in the color and fluorescein images. Can et al. [1] and Tsai et al. [6] extract Y-features while tracing the vessels. They define the Y-feature where three traced lines meet. However, since three lines do not always meet at a unique point, multiple Y-features are often generated in one bifurcation area. In [8], the authors extract Y-features using T- or Y-shape structural element in morphology but the method generate many

false alarms in detection. In [2], Y-features are extracted using an articulated Y-feature model. Initial Y-features candidates are first found using PCA analysis; then at each candidate position, an articulated Y-feature model is adjusted and converged to the bifurcation point in the image using gradient descent. Y-features are matched using mutual information [7] which is very robust to intensity changes across images.



**Figure 1.** Selected images from a sequence of color and fluorescein images.

The next process after feature extraction and matching is global registration. Image sequences of angiograms can be registered sequentially [5][6]. However, if an image is not perfectly registered, this error is propagated to the remaining images. In [2], a global registration is introduced to automatically reduce the registration errors across color and fluorescein images. Global registration is intended to identify the best registration among every pair of images, while minimizing the global registration error using a *minimum spanning tree* (MST) approach. However the authors did not address the problem of selecting the reference frame for the mosaic. Besides, global registration using MST does not guarantee the lowest registration error because MST does not consider the error between the reference frame and other frames.

We propose here instead an algorithm for *provably* optimal registration which gives the lowest registration error from the defined cost function for the multiple images. We extract Y-features using articulated Y-feature model and match them across images using

mutual information. We use the all-pairs shortest path algorithm to find the reference frame for a mosaic and to register other frames optimally.

The rest of the paper is organized as follows: Section 2 describes the process of extracting accurate Y-feature positions and matching Y-features across images. In Section 3, we propose the new algorithm to register a sequence of images optimally. Section 4 discusses the experimental results and a comparison of the performance with other registration algorithms. The last section concludes the paper and outlines future research directions.

## 2. Extracting and Matching Features

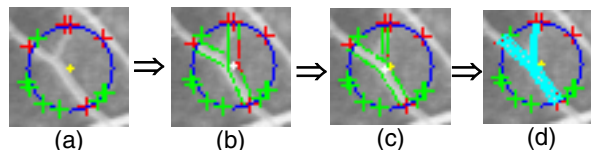
In this section, we briefly present our method for locating Y-features in color and fluorescein images and matching them across images [2]. A Y-feature occurs where three vessels converge. The position of a Y-feature has at least 3 strong responses from 6 different directional Laplacian of Gaussian filter outputs. A set of initial Y-feature positions are found using Principal Components Analysis (PCA) from the 6 filter outputs at every pixel

The considered articulated model for the Y-features has 8 DOF,  $\mathbf{x}=(x,y,\theta_1,\theta_2,\theta_3,W_1,W_2,W_3)$  which include the center position, orientation angle of the three branches attached to the center position, and three widths for each branch. The length of each branch,  $L$ , is fixed. Given the initial position of the Y-feature provided by the estimated initial points, we fit the articulated model using a gradient descent method to minimize the following energy:

$$E(\mathbf{x}) = (-1)^m \frac{1}{2} \sum_{i=1}^3 \int_{-W_i}^{W_i} \int_{-W_i}^{W_i} [I(x_i, y_i)]^2 dl dw - \frac{1}{2} \sum_{i=1}^3 \sum_{w \in \{-W_i, W_i\}} \int_{-W_i}^{W_i} [G(x_i, y_i)]^2 dl \quad (1)$$

where  $i$  is the index of the considered arm.  $I(x_i, y_i)$  is the image intensity in the articulated Y-feature model at  $x_i = x + w \cdot \sin \theta_i + l \cdot \cos \theta_i$ ,  $y_i = y + w \cdot \cos \theta_i + l \cdot \sin \theta_i$  and  $(x, y)$  is the center position of the model.  $G(x_i, y_i)$  is the gradient value along the boundary of the model, and  $m=0$  to find dark vessels,  $m=1$  to detect bright vessels. The model is fitted with the constraint that the angles between branches should not be too close or too far. These constraints can be implemented using Lagrange multipliers for solving the optimization problem with inequality constraints [2]. Figure 2 shows the fitting procedure of a Y-feature.

To match Y-features in one image with those in the other image, the windows enclosing Y-features are matched. The pair with maximum mutual information is selected as a matching pair [2][7].



**Figure 2.** The procedure of fitting an articulated Y-feature model with gradient descent. (a) shows the initial position of Y-feature. Gradient descent method iteratively updates the Y-feature articulated model as shown in (b) and (c); and the converged model is shown in (d).

## 3. Global Registration

In this section, we explain global registration of the multiple images by constructing a graph of the images. For registration of multiple images, we propose an algorithm to find the optimal registration with the lowest registration error by constructing the graph of images where each edge represents the error of pairwise registration. We find the best reference frame and then construct a mosaic by calculating the shortest path from the reference frame to each of the remaining frames.

### 3.1. Pairwise Registration

Using accurately matched pairs of Y-features, two images are registered using an affine transform. The RANSAC [3] method is used to find the inliers among the matched features. The best three matching pairs are selected to obtain the best affine transform, which minimizes the geometric errors of every match:

$$\mathbf{A}_{best} = \arg \text{Min}_{\mathbf{A}} \left[ \frac{1}{2N} \sum_{i=1}^N \left\{ (\mathbf{x}_i - \mathbf{A} \mathbf{y}_i)^2 + (\mathbf{y}_i - \mathbf{A}^{-1} \mathbf{x}_i)^2 \right\} \right] \quad (2)$$

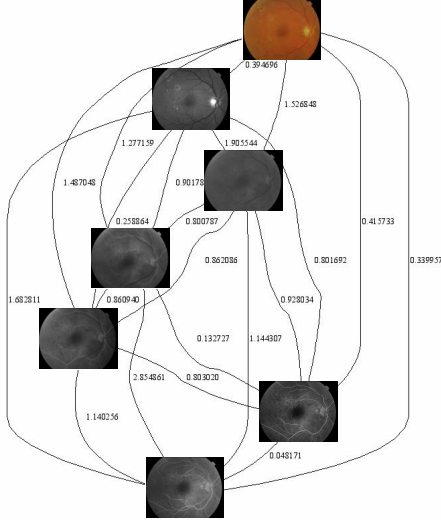
where  $N$  is the number of matching pairs,  $\mathbf{x}_i$  and  $\mathbf{y}_i$  are a matching pair, and  $\mathbf{A}$  is affine transform obtained from selected 3 pairs of Y-features. With the selected affine transform, the geometric error of each matching pairs is computed. Based on the error, the outliers of the matching pairs are identified and removed and the remaining inliers are considered in estimating the affine transform.

### 3.2. Selection of the Reference Frame

The quality of the resulting mosaic depends considerably on which image is selected to be the reference frame. We propose here a method for selecting the reference frame that gives the lowest registration error using graph analysis. To address this problem, we consider a graph-based representation. We construct a complete and undirected graph, where the nodes correspond to the images to be registered and the edges corres

pond to pairwise registration of the images. The cost of each edge is associated to the registration error

obtained by the pairwise registration described in Equation (2). Figure 3 shows an example of a complete graph of images; there are 17 images in this set of data but for clarity, only 7 images are shown for display purpose.



**Figure 3.** Complete graph of a sequence of images.

The optimal registration problem is formulated as finding shortest path from the all nodes to all nodes in this complete and undirected graph. The all-pairs shortest paths are calculated using Floyd-Warshall algorithm [4]. With this algorithm, all shortest paths from a node to any other nodes are obtained. When there are  $n$  images in a sequence, we build an  $n \times n$  size symmetric adjacency matrix  $\mathbf{A}$  where each element represents the geometric error of two images. If there is no connection between two images, the infinite value is assigned for that corresponding element of the adjacency matrix  $\mathbf{A}$ .

```

Procedure Floyd_shortest_path( $n, \mathbf{A}, \mathbf{s}$ )
begin
  for  $k = 1$  to  $n$  do
    for  $i = 1$  to  $n$  do
      for  $j = 1$  to  $n$  do
        if  $\mathbf{A}[i, k] + \mathbf{A}[k, j] < \mathbf{A}[i, j]$  then
          begin
             $\mathbf{A}[i, j] = \mathbf{A}[i, k] + \mathbf{A}[k, j]$ ;
             $\mathbf{s}[i, j] = \mathbf{s}[i, k] + \mathbf{s}[k, j]$ ;
          end
        end
      end
    end
  end Floyd_shortest_path

```

After running this algorithm, the shortest path list from image  $i$  to image  $j$  is saved in the list  $\mathbf{s}[i, j]$ . In the matrix  $\mathbf{A}$ , the accumulated costs for each shortest path are calculated. The  $i$ -th row of matrix  $\mathbf{A}$  indicates the registration errors from the image  $i$  to all other images. Therefore, the values in each row of matrix  $\mathbf{A}$  are summed up and the row with the minimum total

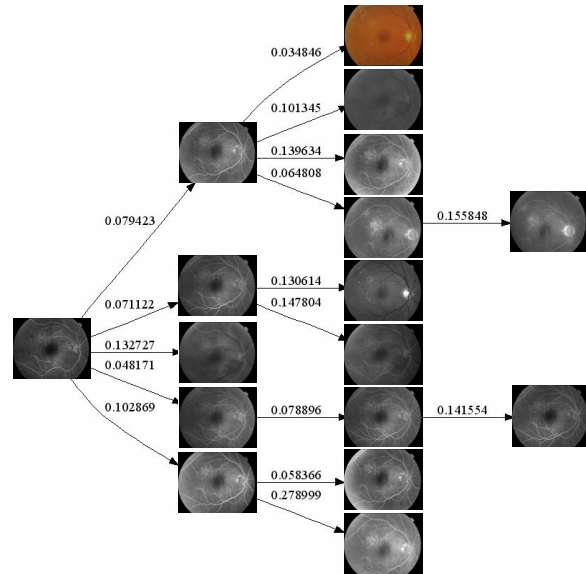
registration errors is selected as the reference frame:

$$\text{Reference\_frame} = \arg \text{Min}_{i \in \{1, \dots, n\}} R_{\text{error}}(i) = \arg \text{Min}_{i \in \{1, \dots, n\}} \sum_{j=1}^n \mathbf{A}[i, j] \quad (3)$$

where  $R_{\text{error}}(i)$  is the total registration error with reference frame  $i$ .

### 3.3. Optimal Registration

After the reference frame is selected, the registration is simply reduced to be the shortest path from reference frame to all other frames as represented by the lists  $\mathbf{s}[i, 1] \dots \mathbf{s}[i, n]$ . This guarantees that the obtained mosaic corresponds to the mosaic with the minimum geometric error across features in the images. The complexity of the algorithm is  $O(n^3)$  where  $n$  is the number of nodes. Figure 4 illustrates the shortest paths from reference frame to other frames.



**Figure 4.** Shortest path from the reference frame to other frames.

## 4. Experimental Results

We conducted experiments on 4 sequences of images, with each sequence having about 15 fluorescein images and one or two color images. For color images, the green channel is extracted to be compared with the gray level images. For each image, the Y-features are extracted using the method described in Section 2. The affine motions for all possible image pairs are calculated. In Figure 5, the final mosaic from the input sequence in Figure 1 is shown with alpha-blending. Figure 1-(e) is selected as a reference frame by the method described in Section 3.2. The other image are registered and blended onto this reference frame based on the shortest path, as illustrated in Figure 5.



**Figure 5.** Final mosaic from the images in Figure 1

To evaluate the performance of registration, a set of matching points on the images were annotated manually. Similarly to the geometric error in Equation (2), the registration error is defined as the average pixel difference of the ground truth pairs after affine transformation. Different from the pairwise error measure in [2], the error here is measured between the reference frame and the other images. In our work, since the error in earlier phase image is propagated and accumulated to the later phase images, the overall error is expected to be higher.

We compared our algorithm with sequential registration and global registration [2] in Table 1. For each algorithm, we calculated the average registration error of Equation (2) on 4 sequences of images with 60 pairs, including very dark images. For sequential and global registration, we evaluated the performance in two ways: (a) when the reference frame is the first frame, and (b) when the reference frame is selected to give the best performance. The result shows that our proposed method achieved more accurate registration compared to the other methods. For time computation, global registration and optimal registration takes almost similar time, since the only difference is in the construction of minimum spanning tree or all pairs' shortest path, and the running time difference is only a few seconds.

**Table 1.** Comparison of average geometric pixel errors using: sequential registration, global registration and optimal registration

Registration Method	Sequential		Global		Optimal
	First	Best	First	Best	
Ave. Error	12.1150	9.3495	6.3313	3.5427	<b>3.3356</b>

## 5. Conclusion

Extracting Y-feature using articulated model and matching them with mutual information provide

accurate matching pairs. The RANSAC based algorithm gives robust affine motion of any two image pair. Constructing a mosaic using all-pairs shortest path algorithm gives optimal solution of pairwise registration from weighted edges, where the weight is defined by a geometric error as described in this paper. All procedures are executed automatically. Our future work will focus on finding a better cost function for pairwise registration such as mutual information. Our proposed optimal registration algorithm is general and not limited on retinal images; therefore we are also exploring the use of this method on general image patches to construct a mosaic.

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